Scanning Array ($\theta = \theta_o$)

$$\psi \Big|_{\theta=\theta_o} = (kd\cos\theta + \beta)\Big|_{\theta=\theta_o} = kd\cos\theta_o + \beta = 0$$

$$\boxed{\beta = -kd\cos\theta_o} \qquad (6\text{-}21)$$

Directivity

N-Element Linear Array

$$D_o = \frac{4\pi U_{\max}}{P_{rad}} = \frac{U_{\max}}{U_o}$$

1. Broadside
2. Ordinary End-Fire

Broadside

$$D_0 = \frac{U_{\max}}{U_0} \simeq \frac{Nkd}{\pi} = 2N\left(\frac{d}{\lambda}\right) \qquad \text{(6-42)}$$

Using $L = (N-1)d$ (6-43)

$$D_0 \simeq 2N\left(\frac{d}{\lambda}\right) \simeq 2\left(1+\frac{L}{d}\right)\left(\frac{d}{\lambda}\right) \qquad \text{(6-44)}$$

For $L >> d$

$$D_0 \simeq 2N\left(\frac{d}{\lambda}\right) = 2\left(1+\frac{L}{d}\right)\left(\frac{d}{\lambda}\right) \simeq 2\left(\frac{L}{\lambda}\right) \qquad \text{(6-44a)}$$

Ordinary End-Fire

$$D_0 = \frac{U_{\max}}{U_0} \simeq \frac{2Nkd}{\pi} = 4N\left(\frac{d}{\lambda}\right) \quad (6\text{-}49)$$

Using $L = (N-1)d$ (6-43)

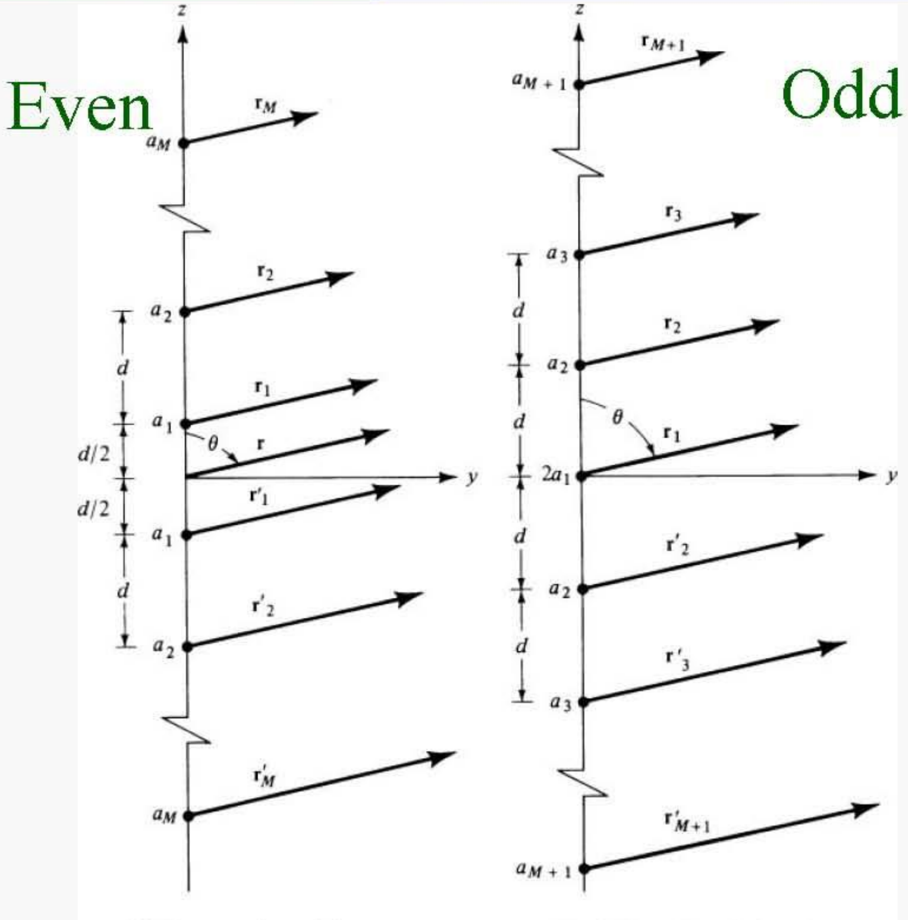
$$D_0 \simeq 4N\left(\frac{d}{\lambda}\right) = 4\left(1+\frac{L}{d}\right)\left(\frac{d}{\lambda}\right) \quad (6\text{-}49a)$$

For $L >> d$

$$D_0 \simeq 4N\left(\frac{d}{\lambda}\right) = 4\left(1+\frac{L}{d}\right)\left(\frac{d}{\lambda}\right) \simeq 4\left(\frac{L}{\lambda}\right) \quad (6\text{-}49b)$$

Nonuniform Arrays

Nonuniform Amplitude Arrays of Even & Odd Number of Elements



(a) Even number of elements (b) Odd number of elements

Fig. 6.19

$N = 2M$ (even)

$$(AF)_{2M} = a_1 e^{j\frac{kd}{2}\cos\theta} + a_2 e^{j\frac{3kd}{2}\cos\theta} + \ldots + a_M e^{j\left(\frac{2M-1}{2}\right)kd\cos\theta}$$

$$+ a_1 e^{-j\frac{kd}{2}\cos\theta} + a_2 e^{-j\frac{3kd}{2}\cos\theta} + \ldots + a_M e^{-j\left(\frac{2M-1}{2}\right)kd\cos\theta}$$

$$2a_1 \cos\left(\frac{kd}{2}\cos\theta\right) \qquad 2a_2 \cos\left(\frac{3kd}{2}\cos\theta\right) \qquad 2a_M \cos\left[\left(\frac{2M-1}{2}\right)kd\cos\theta\right]$$

$$(AF)_{2M} = 2\sum_{n=1}^{M} a_n \cos\left[\frac{(2n-1)}{2}kd\cos\theta\right] \qquad (6\text{-}59)$$

$$(AF)_{2M}\Big|_{\text{norm}} = \sum_{n=1}^{M} a_n \cos\left[\frac{(2n-1)}{2}kd\cos\theta\right] \qquad (6\text{-}59\text{a})$$

$N = 2M+1$ (odd)

$$(AF)_{2M+1} = 2a_1 + a_2 e^{jkd\cos\theta} + \ldots + a_{M+1} e^{j\,M\,kd\cos\theta} + a_2 e^{-jkd\cos\theta} + \ldots + a_{M+1} e^{-j\,M\,kd\cos\theta}$$

$$2a_1 \quad 2a_2\cos(kd\cos\theta) \quad 2a_{M+1}\cos(Mkd\cos\theta)$$

$$(AF)_{2M+1} = 2\sum_{n=1}^{M+1} a_n \cos\left[(n-1)kd\cos\theta\right] \qquad \text{(6-60)}$$

$$(AF)_{2M+1}\Big|_{\text{norm}} = \sum_{n=1}^{M+1} a_n \cos\left[(n-1)kd\cos\theta\right] \qquad \text{(6-60a)}$$

$$\text{Let:}\quad \frac{kd}{2}\cos\theta = \frac{\pi}{\lambda} d\cos\theta = u$$

Normalized Array Factors

$$(AF)_{2M}(even) = \sum_{n=1}^{M} a_n \cos\left[(2n-1)u\right] \quad \text{(6-61a)}$$

$$(AF)_{2M+1}(odd) = \sum_{n=1}^{M+1} a_n \cos\left[2(n-1)u\right] \quad \text{(6-61b)}$$

$$u = \frac{\pi d}{\lambda}\cos\theta \quad \text{(6-61c)}$$

Binomial Expansion

$$(a+b)^n = \frac{a^n b^0}{0!} + n\frac{a^{n-1}b^1}{1!} + n(n-1)\frac{a^{n-2}b^2}{2!} + \cdots$$

$$(1+x)^{m-1} = (+1)^{m-1}\frac{(x)^0}{0!} + (m-1)(+1)^{m-2}\frac{x^1}{1!} + \cdots$$

$$(1+x)^{m-1} = \boxed{1} + \boxed{m-1}\,x + \boxed{\frac{(m-1)(m-2)}{2!}}\,x^2 + \cdots \quad (6\text{-}62)$$

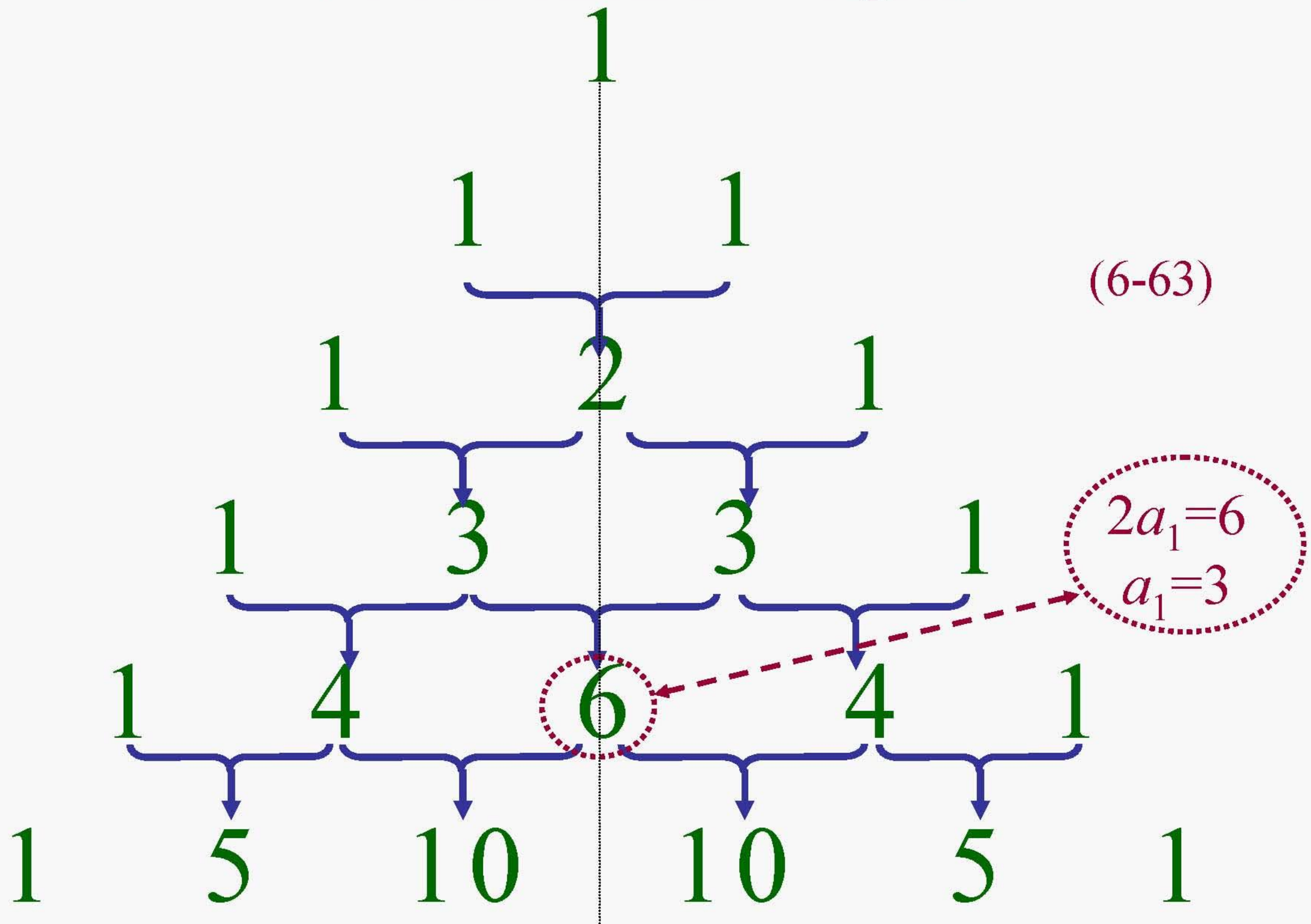
$\underline{m=1}: (1+x)^0 = 1 \Rightarrow$ 1

$\underline{m=2}: (1+x)^1 = 1+(1)x \Rightarrow$ 1 1

$\underline{m=3}: (1+x)^2 = 1+2x+(1)x^2 \Rightarrow$ 1 2 1

$\underline{m=4}: (1+x)^3 = 1+3x+3x^2+(1)x^3 \Rightarrow$ 1 3 3 1

Pascal's Triangle



Example: Binomial ($N = 5$; odd)

				1				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

$$2M+1=5 \Rightarrow M=2$$

$$2a_1=6 \Rightarrow a_1=3$$

$$a_2=4 \qquad a_2=4$$

$$a_3=1 \qquad a_3=1$$

Binomial Design

1. Excitation Coefficients from Pascal's triangle (N = 10 Elements)

$$\underbrace{1}_{a_5}\quad \underbrace{9}_{a_4}\quad \underbrace{36}_{a_3}\quad \underbrace{84}_{a_2}\quad \underbrace{126}_{a_1}\quad \underbrace{126}_{a_1}\quad \underbrace{84}_{a_2}\quad \underbrace{36}_{a_3}\quad \underbrace{9}_{a_4}\quad \underbrace{1}_{a_5}$$

$$\left.\begin{aligned} a_1 &= 126 \\ a_2 &= 84 \\ a_3 &= 36 \\ a_4 &= 9 \\ a_5 &= 1 \end{aligned}\right\} \text{Excitation Coefficients}$$

2. Spacing

A. For No Side Lobes, Select

$$d \leq \lambda / 2$$

B. For No "Grating" Lobes, Select

$$d < \lambda$$

Array Factor Power Patterns for a 10-Element Broadside Binomial Array

$\underline{N = 10}$

$d = \lambda/4,$

$\lambda/2,$

$3\lambda/4,$

λ

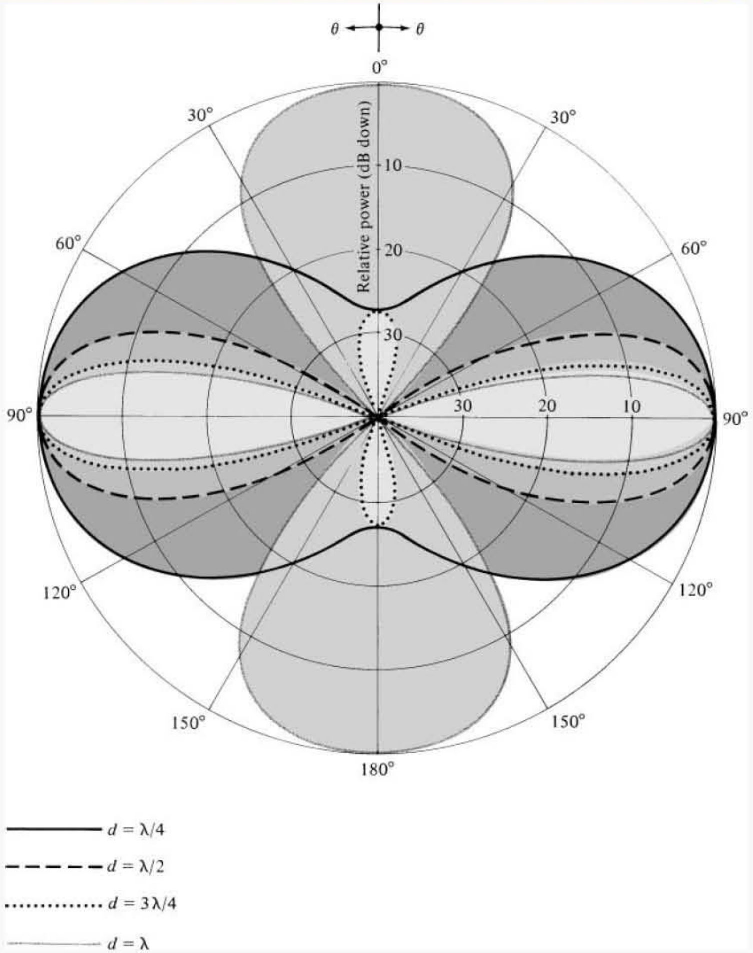


Fig. 6.20

Design Procedure $(d = \lambda/2)$

$$\text{HPBW} \cong \frac{1.06}{\sqrt{N-1}} = \frac{1.06}{\sqrt{2L/\lambda}} = \frac{0.75}{\sqrt{L/\lambda}} \quad \text{(6-64)}$$

$$D_o = \frac{2}{\int_o^{\pi} \left[\cos\left(\frac{\pi}{2}\cos\theta\right)\right]^{2(N-1)} \sin\theta d\theta} \quad \text{(6-65)}$$

$$D_o = \frac{(2N-2)(2N-4)\ldots 2}{(2N-3)(2N-5)\ldots 1} \quad \text{(6-65a)}$$

$$D_o \cong 1.77\sqrt{N} = 1.77\sqrt{1+\frac{2L}{\lambda}} \quad \text{(6-65a)}$$

Example 6.8:

$$N = 10,\ d = \lambda/2,\ \text{Binomial},\ \text{HPBW} = ?,\ D_0 = ?$$

Solution:

$$\text{HPBW} = \frac{1.06}{\sqrt{10-1}} = 0.353\ rad = 20.23^\circ$$

$$\text{HPBW(exact)} = 20.5^\circ$$

$$D_0 = \frac{18(16)(14)(12)(10)(8)(6)(4)(2)}{17(15)(13)(11)(9)(7)(5)(3)(1)} = 5.392 = 7.32\ \text{dB}$$

$$D_0 \simeq 1.77\sqrt{10} = 5.597 = 7.48\ \text{dB}$$

$$D_0 = 5.5392 = 7.32\ \text{dB}\ \ \text{Computer Program}$$